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STRESS INDICES AND FLEXIBILITY FACTORS FOR 90-DEGREE PIPING ELB--ETC(U)
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STRESS INDICES AND FLEXIBILITY FACTORS FOR 90-DEGREE PIPING ELBOWS WITH STRAIGHT PIPE EXTENSIONS

DTNSRDC/CMLD-82/05

DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

STRESS INDICES AND FLEXIBILITY FACTORS FOR 90-DEGREE PIPING ELBOWS WITH STRAIGHT PIPE EXTENSIONS

by

A. J. Quezon and G. C. Everstine

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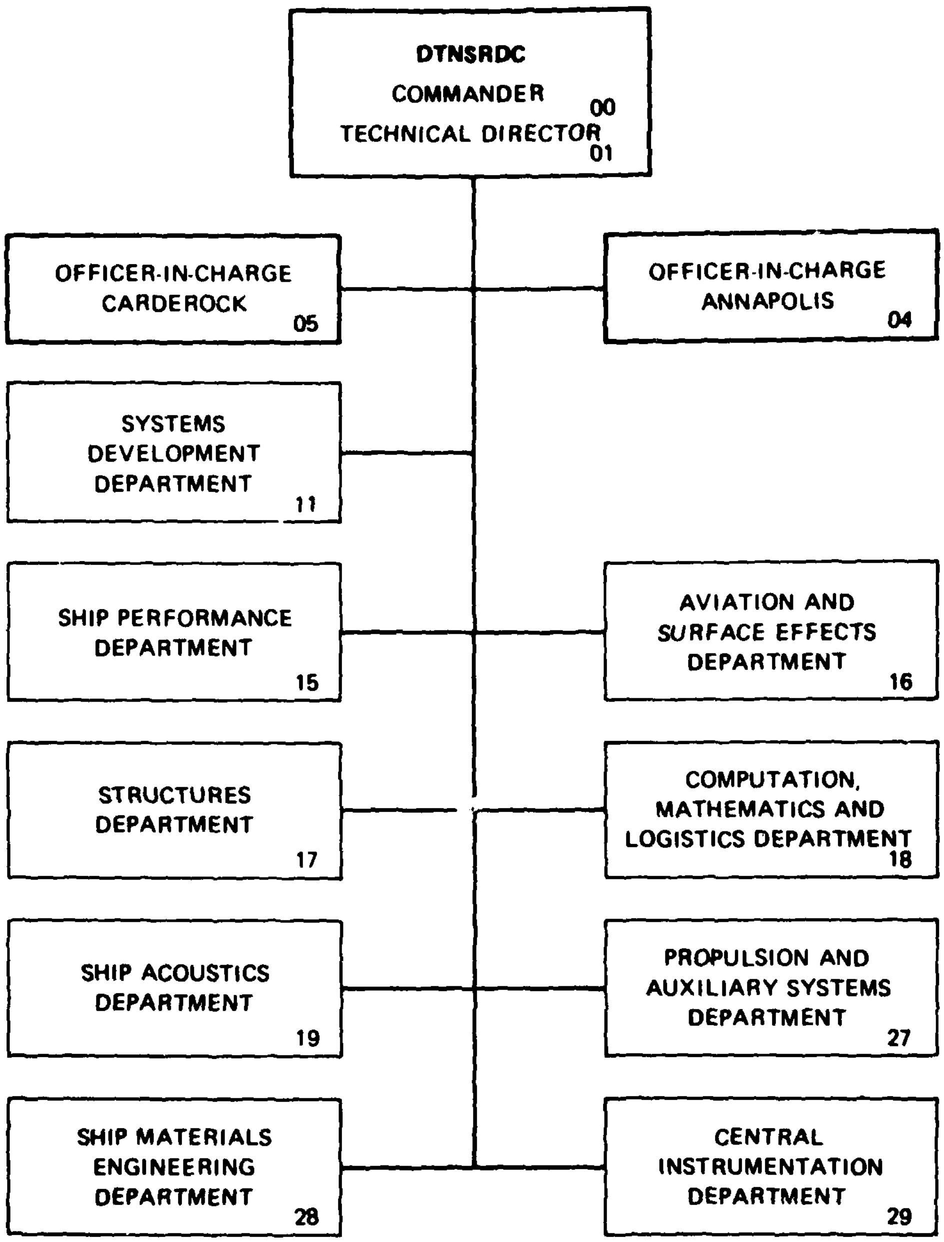
COMPUTATION, MATHEMATICS, AND LOGISTICS DEPARTMENT
DEPARTMENTAL REPORT

February 1982

DTNSRDC/CMLD-82/05

MAJOR DTNSRDC ORGANIZATIONAL COMPONENTS



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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER DTNSRDC/CMLD-82/05	2. GOVT ACCESSION NO. AD-A112050	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) STRESS INDICES AND FLEXIBILITY FACTORS FOR 90-DEGREE PIPING ELBOWS WITH STRAIGHT PIPE EXTENSIONS		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) A. J. Quezon and G. C. Everstine		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS David W. Taylor Naval Ship Research and Development Center Bethesda, Maryland 20084		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element 63561N Task Area S0348001 Work Unit 1-2740-163
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE February 1982
		13. NUMBER OF PAGES 53
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Piping	Finite Element	Flexibility Factor
Elbow	NASTRAN	
Bend	Stress Index	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A finite element parameter study was performed on a wide variety of 90-degree piping elbows having straight pipe extensions to determine the sensitivity of stress indices and flexibility factors to the length of the pipe extensions. Both moment and force loadings were considered. It was found that stress indices are generally insensitive to the type of load (moment or force). Flexibility factors are sensitive to load type only

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for short pipe extensions. Flexibility factors for moments generally exceed those for forces. Stress indices are sensitive to length of pipe extension only for $\lambda < 0.35$, where λ is the bend characteristic parameter. Flexibility factors are sensitive to length of pipe extension over the entire range of elbow parameters considered. The "critical length" of straight pipe extension (defined as the minimum length of pipe for which the elbow stress index is insensitive to further increases in pipe length) was found to be about three pipe diameters for $\lambda < 0.35$ and one diameter for $\lambda > 0.35$. A comparison of the finite element results to those of a torus program called ELBOW indicated that ELBOW is generally not adequate for predicting stress indices and flexibility factors for 90-degree elbows with straight pipe extensions.

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ABSTRACT

A finite element parameter study was performed on a wide variety of 90-degree piping elbows having straight pipe extensions to determine the sensitivity of stress indices and flexibility factors to the length of the pipe extensions. Both moment and force loadings were considered. It was found that stress indices are generally insensitive to the type of load (moment or force). Flexibility factors are sensitive to load type only for short pipe extensions. Flexibility factors for moments generally exceed those for forces. Stress indices are sensitive to length of pipe extension only for $\lambda < 0.35$, where λ is the bend characteristic parameter. Flexibility factors are sensitive to length of pipe extension over the entire range of elbow parameters considered. The "critical length" of straight pipe extension (defined as the minimum length of pipe for which the elbow stress index is insensitive to further increases in pipe length) was found to be about three pipe diameters for $\lambda < 0.35$ and one diameter for $\lambda > 0.35$. A comparison of the finite element results to those of a torus program called ELBOW indicated that ELBOW is generally not adequate for predicting stress indices and flexibility factors for 90-degree elbows with straight pipe extensions.

ADMINISTRATIVE INFORMATION

This work was sponsored by the Naval Sea Systems Command in Program Element/Task Area 63561N/S0348001, Task 21302, Work Unit 1-2740-163. Naval Sea Systems Command cognizant program manager is Dr. F. Ventriglio (NAVSEA 05R).

INTRODUCTION

Common practice in the design of shipboard piping systems is to compute stresses and flexibilities for piping elbows using elementary beam theory for straight beams and applying correction factors called stress

indices and flexibility factors. The correction factors are generally calculated by assuming that the elbow behaves as if it were part of a complete torus, for which analytical solutions for stresses and flexibilities are available. This approach thus accounts for the bend radius, the pipe diameter, the wall thickness, the material properties, and the internal pressure (a nonlinear effect when combined with other loads). However, the approach ignores such considerations as the bend angle and end conditions, which can include flanges, straight pipe extensions, or other elbows.

Since the inclusion of these considerations rules out the use of analytic solutions, the only way to compute stresses and flexibilities in such cases is to use an approximate numerical approach such as the finite element method. The finite element method is well-established in general and has, in particular, been verified as suitable for predicting the static behavior of piping elbows and tees [1-3].*

The overall aim of this work is to determine the flexibilities and stresses in piping elbows and bends in the configurations commonly used on naval ships. This report addresses in particular the end condition problem by analyzing in some detail 90-degree elbows with straight pipe extensions of various lengths.

The general approach is to analyze a selection of elbows of a variety sufficient (1) to determine the sensitivity of the elbow stress indices and flexibility factors to the length of straight pipe extensions, and (2) to determine the minimum length of straight pipe extension (the "critical length") for which the stress indices are considered insensitive to further increases in pipe length.

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* A complete listing of references is given on page 51.

The volume of data generated by these analyses is sufficiently complex that only preliminary conclusions will be drawn from the data at this time. A follow-up report will present a more complete analysis of the data presented here and develop usable design equations, formulas, or curves.

DEFINITIONS

The stress index c for an elbow is defined [4,5] as the ratio of the maximum stress intensity for the elbow to the maximum bending stress in a straight pipe having the same cross section. *The stress intensity S is* defined as twice the maximum shear stress in the elbow for a given loading condition. Thus the maximum stress intensity is the maximum of

$$
\begin{aligned}
S_1 &= |\sigma_1 - \sigma_2| = \{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2\}^{\frac{1}{2}} \\
S_2 &= |\sigma_1 - \sigma_3| = |(\sigma_x + \sigma_y) + S_1|/2 \\
S_3 &= |\sigma_2 - \sigma_3| = |(\sigma_x + \sigma_y) - S_1|/2
\end{aligned}
\qquad (1)
$$

for any arbitrary orientation of an applied moment vector M, where, for the two-dimensional state of stress that occurs in thin-walled piping elbows,

σ_1 = maximum principal stress

σ_2 = minimum principal stress

σ_3 = 0 (the third principal stress in 3-D elasticity)

σ_x, σ_y = normal stresses in longitudinal and circumferential directions

τ_{xy} = shear stress

The calculation of S_{max} (the maximum of S_1, S_2, and S_3) requires a search over all possible orientations of the moment vector M. Hence, the stress index c is given by

$$c = S_{max}/\sigma_{nom} \tag{2}$$

where σ_{nom} is the nominal stress for the corresponding straight pipe as predicted by elementary beam theory:

$$\sigma_{nom} = M/Z \tag{3}$$

where Z is the section modulus.

For internal pressure loading,

$$c = \sigma_{max}/\sigma_{nom} \tag{4}$$

where σ_{max} is the maximum stress in the elbow subjected to internal pressure. The nominal stress σ_{nom} is taken as the maximum stress occurring in a cylindrical pressure vessel due to an internal pressure load:

$$\sigma_{nom} = pr/t \tag{5}$$

where p is the applied internal pressure, r is the mean pipe radius, and t is the wall thickness.

The flexibility factor k for a piping component (e.g., an elbow) is defined as the ratio of a relative rotation of that component to a nominal rotation:

$$k = \theta_{ab}/\theta_{nom} \tag{6}$$

where θ_{ab} = rotation of end "a" of the piping component relative to end "b" of that component due to a moment loading M, and in the direction of M

θ_{nom} = nominal rotation of an equal length of straight pipe due to the moment M

For elbows, the nominal rotation is computed using beam theory, in which case

$$\theta_{nom} = ML/EI \tag{7}$$

for inplane and out-of-plane moments, and

$$\theta_{nom} = ML/GJ \tag{8}$$

for torsional moments, where

M = applied moment load

L = arc length of centerline of elbow

E = Young's modulus of material

G = shear modulus of material

I = moment of inertia of cross section

J = torsional constant of cross section (equal to the polar moment of inertia for circular cross sections)

For 90-degree elbows, $L=\pi R/2$, where R is the bend radius.

The critical length of pipe extension for an elbow is defined here as the minimum length of straight pipe extension for which the stress index is considered insensitive to further increases in pipe length. This sensitivity results from the reduced ability of the elbow to ovalize due to the presence of straight pipes and flanges. At some length of pipe extension (the critical length), additional end effects due to the restriction of ovalization no longer occur.

The bend characteristic parameter λ, a dimensionless parameter widely used in elbow design, is defined as

$$\lambda = tR/r^2\sqrt{1-\nu^2} \tag{9}$$

where t = wall thickness

R = bend radius

r = mean pipe radius

ν = Poisson's ratio of material

The nondimensional bend radius γ is defined as

$$\gamma = R/r \tag{10}$$

The internal pressure loading parameter ψ is defined as

$$\psi = pR^2/Ert \tag{11}$$

where p is the internal pressure.

SCOPE OF STUDY

Table 1 summarizes the piping elbows of interest to naval piping designers. (The data in this table were compiled by Mr. L.M. Kaldor of the Machinery Stress Analysis Branch (Code 2744) of the David W. Taylor Naval Ship R&D Center, Annapolis, Maryland.) Included are elbows of 19 nominal pipe sizes, four bend radii, and three materials. The data in the table represent a total of 204 different elbows, a number which must be reduced if the parameter study is to be reasonably manageable.

The information in Table 1 can also be displayed in nondimensional form (Table 2) by dividing all length dimensions by the mean pipe radius r for each elbow. A graphical display is more useful still. Since the two key geometrical parameters defining an elbow are its nondimensional wall thickness and bend radius, we can use these two parameters to construct a figure (Figure 1) which locates all elbows of interest. In Figure 1, a dot is entered for

TABLE 1 - DIMENSIONS OF PIPING ELBOWS

Nominal Pipe Size (NPS)	Outside Diameter (in.)	Min. Wall Thickness (in.)			Bend Radius (in.)			
		CA715 700 psi	CA719 1050 psi	In625 1050 psi	SR	LR	3D	5D
1/4	.540	.065	.106	.012*	-	-	-	1.25
3/8	.675	.072	.110	.014*	-	-	-	1.875
1/2	.840	.072	.115	.018*	-	1.5	1.5	2.5
3/4	1.050	.083	.121	.022*	-	1.125	2.25	3.75
1	1.315	.095	.129	.028*	1	1.5	3	5
1 1/4	1.660	.095	.139	.035*	1.25	1.875	3.75	6.25
1 1/2	1.900	.109	.146	.040*	1.5	2.25	4.5	7.5
2	2.375	.120	.160	.050*	2	3	6	10
2 1/2	2.875	.134	.175	.060*	2.5	3.75	7.5	12.5
3	3.500	.165	.193	.073	3	4.5	9	15
3 1/2	4.000	.180	.208	.083	3.5	5.25	10.5	17.5
4	4.500	.203	.222	.093	4	6	12	20
5	5.563	.220	.253	.115	5	7.5	15	25
6	6.625	.259	.285	.137	6	9	18	30
8	8.625	.340	.343	.179	8	12	24	40
10	10.750	.380	.405	.223	10	15	30	50
12	12.750	.454	.464	.264	12	18	36	60
14	14.000	.473	.501	.290	14	21	42	70
16	16.000	.534	.559	.331	16	24	48	80
Poisson's ratio		.294	.294	.309				
Young's modulus		22.E6	22.E6	30.E6 psi				

* Thickness may be greater due to welding considerations.

TABLE 2 - NONDIMENSIONAL ELBOW GEOMETRIC DATA

NPS	Mean Rad. (in.)*			Nondim. Thick.			Nondimensional Bend Radius (R/r)											
	a	b	c	t_a/a	t_b/b	t_c/c	SR/a	LR/a	3D/a	5D/a	SR/b	LR/b	3D/b	5D/b	SR/c	LR/c	3D/c	5D/c
1/4	.238	.217	.264	.273	.488	.045	-	-	-	5.25	-	-	-	5.76	-	-	-	4.73
3/8	.302	.283	.331	.283	.389	.042	-	-	-	6.21	-	-	-	6.63	-	-	-	5.66
1/2	.384	.363	.411	.188	.317	.044	-	3.91	3.91	6.51	-	4.13	4.13	6.89	-	3.65	3.65	6.08
3/4	.484	.465	.514	.171	.260	.043	-	2.32	4.65	7.75	-	2.42	4.84	8.06	-	2.19	4.38	7.30
1	.610	.593	.644	.156	.218	.043	1.64	2.46	4.92	8.20	1.69	2.53	5.06	8.43	1.55	2.33	4.66	7.76
1 1/4	.783	.761	.813	.121	.183	.043	1.60	2.40	4.79	7.98	1.64	2.46	4.93	8.21	1.54	2.31	4.61	7.69
1 1/2	.896	.877	.930	.122	.166	.043	1.67	2.51	5.02	8.37	1.71	2.57	5.13	8.55	1.61	2.42	4.84	8.06
2	1.128	1.108	1.163	.106	.144	.043	1.77	2.66	5.32	8.87	1.81	2.71	5.42	9.03	1.72	2.58	5.16	8.60
2 1/2	1.371	1.350	1.408	.098	.130	.043	1.82	2.74	5.47	9.12	1.85	2.78	5.56	9.26	1.78	2.66	5.33	8.88
3	1.668	1.654	1.714	.099	.117	.043	1.80	2.69	5.40	8.99	1.81	2.72	5.44	9.07	1.75	2.63	5.25	8.75
3 1/2	1.910	1.896	1.959	.094	.110	.042	1.83	2.75	5.50	9.16	1.85	2.77	5.54	9.23	1.79	2.68	5.36	8.93
4	2.149	2.139	2.204	.094	.104	.042	1.86	2.79	5.58	9.31	1.87	2.81	5.61	9.35	1.82	2.72	5.45	9.07
5	2.672	2.655	2.724	.082	.095	.042	1.87	2.81	5.61	9.36	1.88	2.82	5.65	9.42	1.84	2.75	5.51	9.18
6	3.183	3.170	3.244	.081	.090	.042	1.89	2.83	5.66	9.43	1.89	2.84	5.68	9.46	1.85	2.77	5.55	9.25
8	4.143	4.141	4.223	.082	.083	.042	1.93	2.90	5.79	9.65	1.93	2.90	5.80	9.66	1.89	2.84	5.68	9.47
10	5.185	5.173	5.264	.073	.078	.042	1.93	2.89	5.79	9.64	1.93	2.90	5.80	9.67	1.90	2.85	5.70	9.50
12	6.148	6.143	6.243	.074	.076	.042	1.95	2.93	5.86	9.76	1.95	2.93	5.86	9.77	1.92	2.88	5.77	9.61
14	6.764	6.750	6.855	.070	.074	.042	2.07	3.10	6.21	10.35	2.07	3.11	6.22	10.37	2.04	3.06	6.13	10.21
16	7.733	7.721	7.835	.069	.072	.042	2.07	3.10	6.21	10.35	2.07	3.11	6.22	10.36	2.04	3.06	6.13	10.21

* Letters a, b, and c refer to materials CA715, CA719, and In625, respectively.

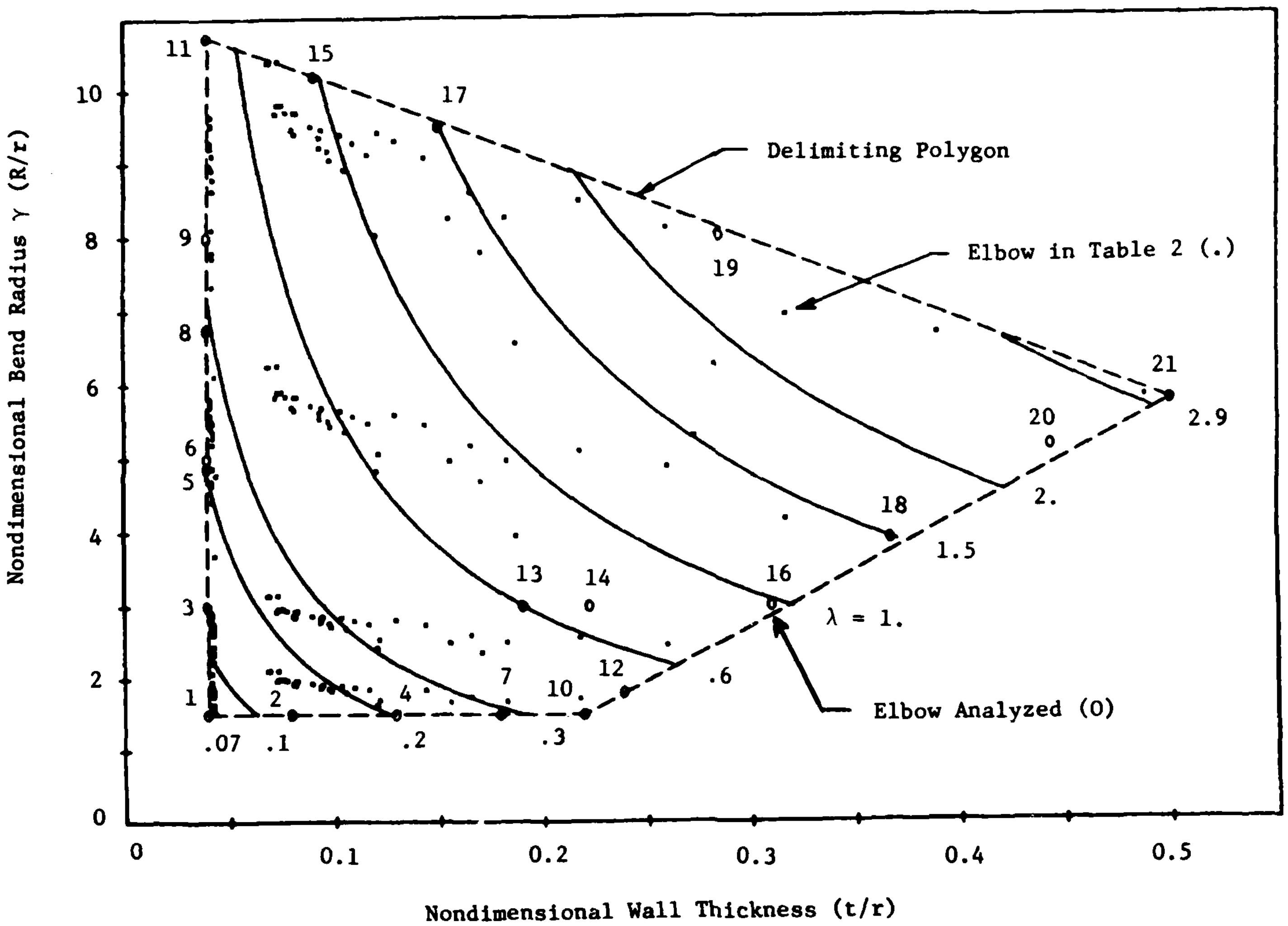

Figure 1 - Pictorial Representation of Nondimensional Elbow Geometric Data

each of the 204 elbows in Tables 1 and 2. The figure is useful because it indicates the ranges of interest for the geometrical parameters t/r and R/r.

The collection of dots in Figure 1 has also been enclosed by a dashed polygon. We expect that the range of behavior (stresses and flexibilities) of elbows can be deduced by studying primarily elbows lying on the periphery of the polygon. Moreover, the elbows actually analyzed would not have to be real elbows appearing in Tables 1 and 2, but merely a suitable collection of elbows defined by parameter pairs (t/r and R/r) lying on the polygon in Figure 1.

Also shown in Figure 1 are several curves of constant λ, the bend characteristic parameter, assuming a Poisson's ratio of 0.3, which is typical for the materials listed in Table 1. These curves are useful for observing the range of λ represented by the elbows in Table 1 and for verifying the extent to which certain elbow behavior depends only on λ, as predicted by the idealized theory for tori.

This study will therefore concentrate on elbows defined by the periphery of the polygon in Figure 1. In addition, only 90-degree elbows with straight pipe extensions will be considered. Elbows with other bend angles and end conditions will be studied in future efforts.

For moment and force loadings, the internal pressure in the pipe is assumed to be zero throughout this report. To assume otherwise introduces a nonlinear effect which would greatly complicate the analyses. However, because the nonzero pressure case is of considerable interest, it will be pursued in future work.

APPROACH

Elbows defined by the parameter pairs t/r and R/r were chosen primarily

from the periphery of the polygon in Figure 1. For each elbow chosen, finite element analyses were performed by the NASTRAN structural analysis computer program in order to compute flexibility factors and stress indices for various lengths of straight pipe extension varying from near zero to three or four pipe diameters.

The elbow configuration modeled consists of a 90-degree elbow with straight pipe extensions attached to each end of the elbow, as shown in Figure 2. The end of one pipe extension was fixed. The end of the other pipe extension was terminated with a rigid flange. Applied loads consisted of internal pressure as well as the six possible forces and moments applied to the flange at the free end. For this study, internal pressure loads are not combined with either force or moment loads.

The finite element results were also compared to results obtained by a fast-running computer program called ELBOW [6] which is used by some piping designers. Program ELBOW uses analytical methods to compute flexibility factors and stress indices for elbows idealized as endless toroidal sections. For zero internal pressure (ψ=0), ELBOW's calculation of the flexibility factor depends only on the bend characteristic parameter λ.

For flexibility factors only, the finite element results were also compared to flexibility factors computed according to the current ASME code [7], which, for zero internal pressure, uses the relation

$$k = 1.65/h \tag{12}$$

where

$$h = tR/r^2 \tag{13}$$

and a k calculated to be less than unity is taken as unity.

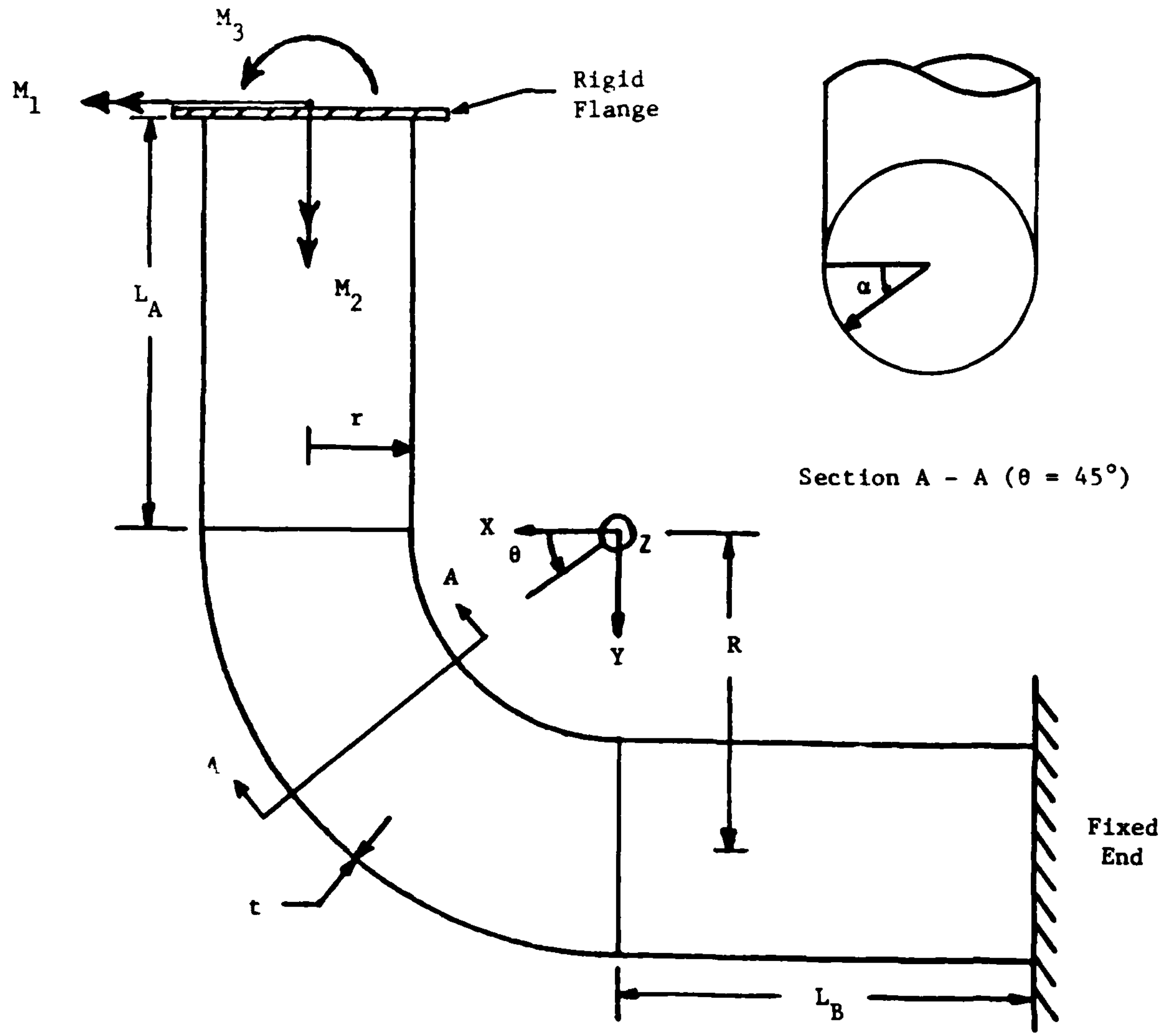

Figure 2 - Geometry and Configuration of Piping Elbow

For internal pressure loading, the finite element results for the stress index c were also compared to the frequently used analytical expression

$$c = (2\gamma - 1)/(2\gamma - 2) \tag{14}$$

presented in the Kellogg book [5], where γ is the nondimensional bend radius (R/r). Since for our elbows γ ranges from about 1.5 to 11, Equation (14) indicates that the internal pressure stress index should be between 1 and 2.

A typical finite element model of an elbow and the two straight pipe extensions is shown in Figure 3. By symmetry, only half of the circumference of the elbow cross section need be modeled. The elbow and pipe extensions were modeled using NASTRAN's two-dimensional quadrilateral QUAD2 plate element with aspect ratios averaging near unity in the elbow region and about two near the ends of the pipe extensions. All models used 12 elements in the circumferential direction.

To compute flexibility factors, the average rotations of the cross sections at each end of the elbow were required. These averages were obtained in each cross section of interest by defining in that cross section an imaginary center point which was connected to the points on the circumference by beam elements flexible enough not to contribute significantly to the stiffness of the model.

A special purpose finite element data generator was written to automate completely the preparation of the NASTRAN input data decks so that the specification of only a few parameters was required to analyze a particular case.

Poisson's ratio was fixed at 0.3 for all analyses to eliminate the material constants as parameters in the study. This assumption has an insignificant effect on the solutions obtained, since stresses and

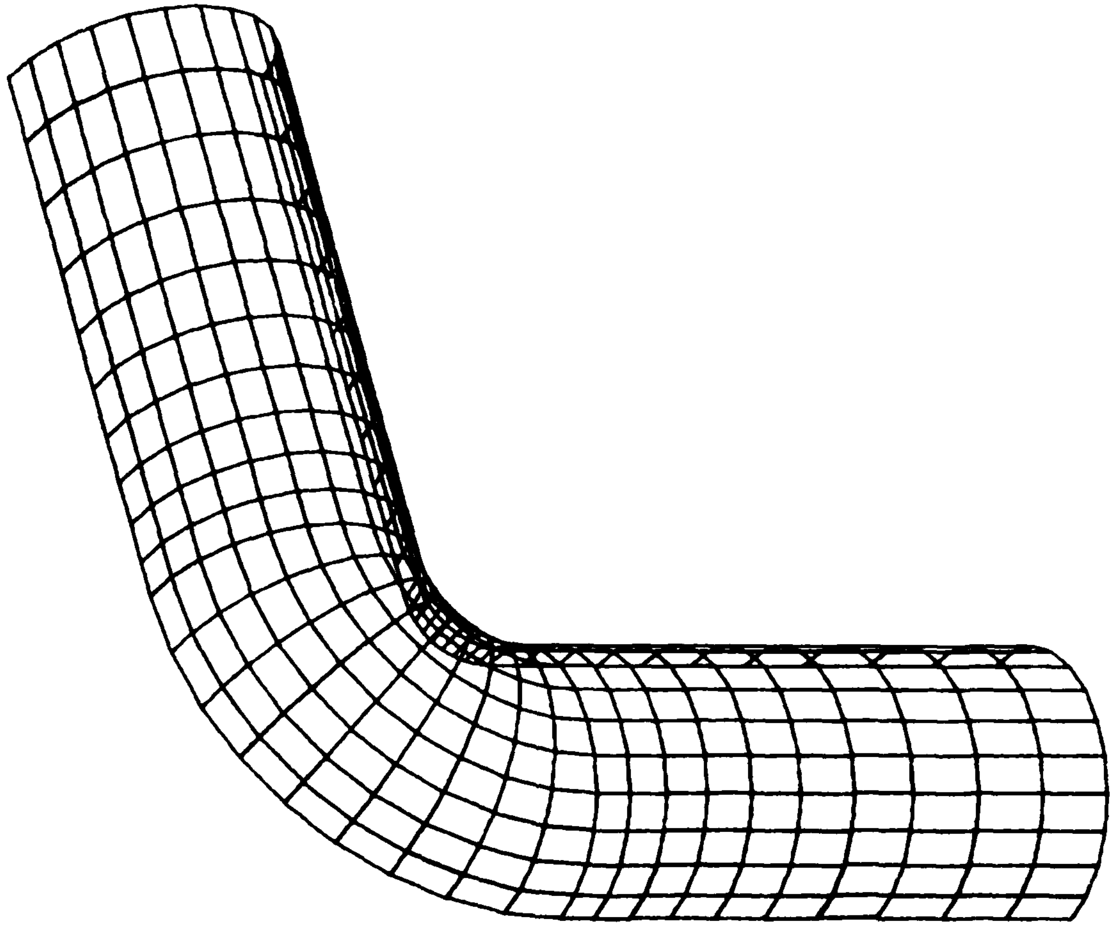

Figure 3 - Typical Finite Element Model of Piping Elbow

flexibilities are very insensitive to small changes in Poisson's ratio.

PRESENTATION AND DISCUSSION OF RESULTS

Finite element analyses were performed for 21 elbows with straight pipe extensions of various lengths. The geometric data for these 21 elbows are plotted in Figure 1. In most cases, the pipe extensions placed at each end of the elbow were of equal length. For each of the 21 elbows, the number of analyses performed (in order to vary the length of pipe extensions) ranged from six to 13, the average being about 7.5. The total number of analyses performed was 157. The stress index and flexibility factor results for all these analyses are tabulated in detail in the Appendix.

Although our original purpose in performing these calculations was to derive usable design equations, formulas, and curves, the complexity of the resulting data (tabulated in the Appendix) makes this a formidable task. This report, which is therefore a little less ambitious, will instead summarize some of the key results indicated by the data. All observations in this section are based on data in the Appendix.

We first consider the influence of the type of loading (moment or force) on the results. The applied forces were chosen so as to yield the same bending moment at the elbow middle, where stresses are generally the greatest. The tables in the Appendix show that stress indices depend very little on whether the applied load is a force or moment, particularly when the pipe extensions are long. The flexibility factors are only slightly more sensitive than the stress indices to load type for elbows with long pipe extensions. However, flexibility factors are quite sensitive to load type for elbows with short pipe extensions. Flexibility factors due to moment loadings

generally exceed those due to force loadings.

In retrospect, this independence of stress index with load type can perhaps be explained by noting that the index depends only on the solution in the middle of the elbow, which is always (for 90-degree elbows) sufficiently far from the ends so as not to be significantly affected by the load type. The elbow flexibility, in contrast, depends on the solution over the entire length of the elbow and hence might be expected to be more sensitive than stress index to load type.

A second question which arises is: How sensitive are stresses and flexibilities to the length of pipe extensions? A related question is: How long do the pipe extensions have to be in order for elbow stresses and flexibilities to be independent of further changes in length? In Figure 4 the dependence of stress index on length of straight pipe extension (equal at both ends) is plotted for various elbows. Figure 4 indicates that stress indices are very sensitive to length of pipe extension for small λ and insensitive for large λ. It appears that the transition from sensitivity to insensitivity occurs near $\lambda = 0.35$. Figure 4 also indicates that the critical length of straight pipe extensions, based on stress indices, is about three pipe diameters for $\lambda < 0.35$ and one pipe diameter for $\lambda > 0.35$.

Table 3 summarizes the stress index results for the 21 elbows with equal length pipe extensions three diameters long. The actual stress indices, which are generally greatest with long (rather than short) pipe extensions, are grossly over-predicted in general by the idealized program ELBOW. For internal pressure loading, the classical relation, Equation (14), is excellent.

Flexibility factors for moment loadings exhibit a sensitivity to length

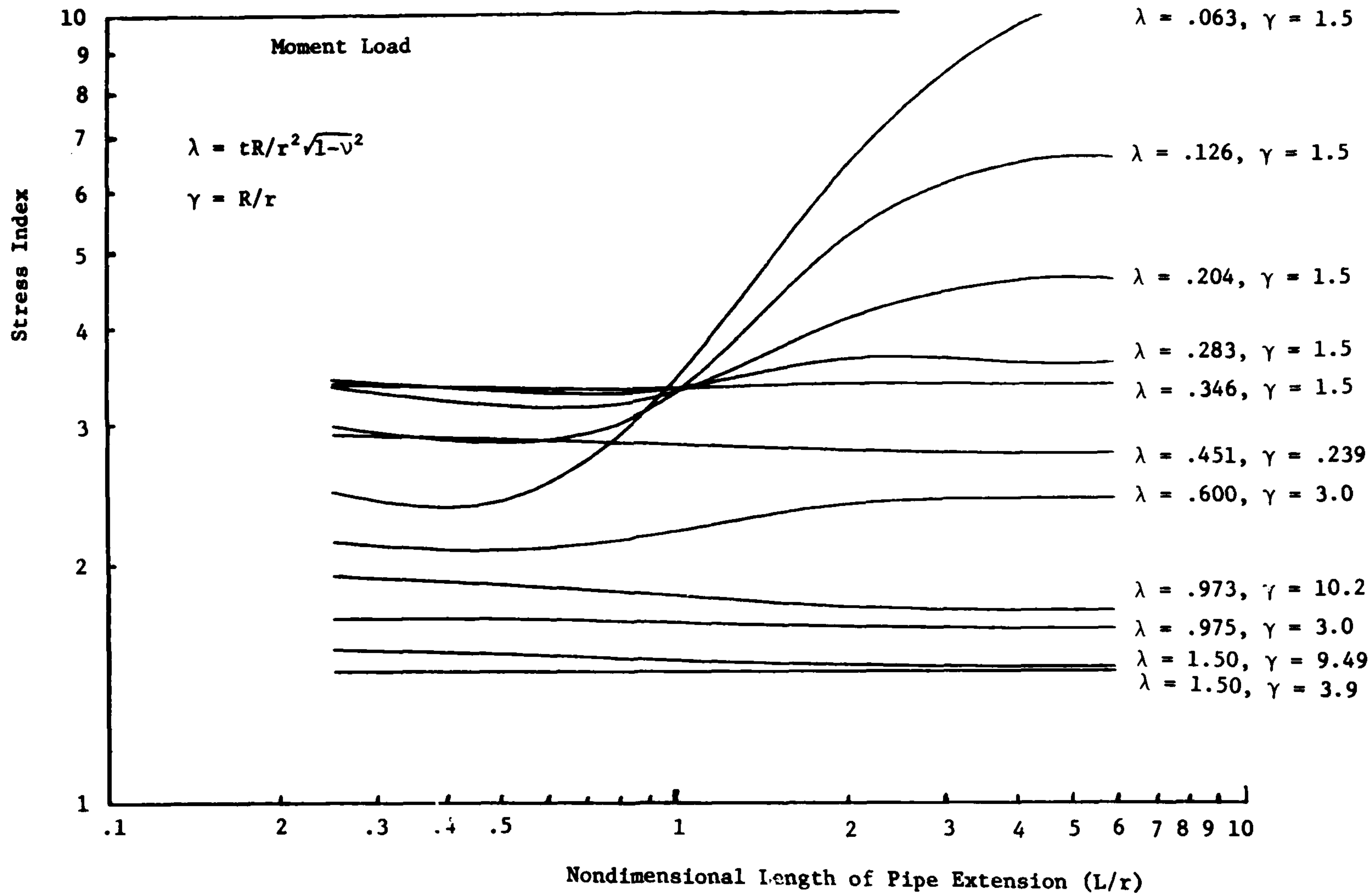

Figure 4 - Sensitivity of Stress Index to Length of Pipe Extension

TABLE 3 - STRESS INDICES FOR ELBOWS HAVING EQUAL LENGTH STRAIGHT PIPE EXTENSIONS OF LENGTH THREE DIAMETERS

	λ	γ	NASTRAN		ELBOW	Internal Pressure	
			Moment	Force		NASTRAN	Kellogg
1	.063	1.50	10.2	10.0	14.5	1.93	2.00
2	.126	1.50	6.55	6.41	9.28	1.89	2.00
3	.126	3.00	7.13	6.96	8.56	1.23	1.25
4	.204	1.50	4.61	4.52	6.82	1.82	2.00
5	.204	4.88	5.39	5.21	5.98	1.12	1.13
6	.210	5.00	5.31	5.18	5.87	1.12	1.13
7	.283	1.50	3.62	3.68	5.53	1.77	2.00
8	.283	6.75	4.39	4.26	4.70	1.08	1.09
9	.335	8.00	3.90	3.81	4.14	1.07	1.07
10	.346	1.50	3.39	3.45	4.88	1.74	2.00
11	.451	10.8	3.10	3.02	3.29	1.04	1.05
12	.451	1.80	2.77	2.83	3.94	1.56	1.63
13	.600	3.00	2.43	2.32	2.93	1.25	1.25
14	.700	3.00	2.12	2.04	2.61	1.25	1.25
15	.973	10.2	1.67	1.71	2.00	1.05	1.05
16	.975	3.00	1.76	1.80	2.13	1.26	1.25
17	1.50	9.49	1.47	1.45	1.71	1.06	1.06
18	1.50	3.90	1.49	1.54	1.75	1.19	1.17
19	2.39	8.00	1.27	1.25	1.52	1.07	1.07
20	2.39	5.15	1.32	1.37	1.52	1.14	1.12
21	3.01	5.75	1.28	1.33	1.45	1.12	1.11

of straight pipe extension similar to that of the stress indices, except that the transition from sensitivity (small λ) to insensitivity (large λ) occurs at about $\lambda = 1.0$. For flexibility factors calculated from force loadings, there is no clear region of insensitivity. Two plots illustrating this sensitivity are shown in Figures 5 and 6. These two figures indicate that elbows become stiffer as the pipe extensions shorten, probably because the flanges inhibit ovalization of the cross section.

Table 4 summarizes the flexibility factor results for the 21 elbows with equal length pipe extensions three diameters long. There is considerable variation between inplane and out-of-plane flexibility factors, neither of which are predicted very well in general by the idealized approach used in the ELBOW program; it does not distinguish between inplane and out-of-plane moments in calculating the flexibility factor. The data listed in Table 4 are also shown graphically in Figure 7.

CONCLUSIONS

The following conclusions apply to 90-degree piping elbows with straight pipe extensions terminated by rigid flanges.

Stress indices are generally insensitive to whether the applied load is a force or a statically equivalent moment. For elbows with long pipe extensions at each end, flexibility factors are only slightly more sensitive than the stress indices to load type. However, flexibility factors are quite sensitive to load type when the pipe extensions are short. Flexibility factors for moment loadings generally exceed those for force loadings.

Stress indices are very sensitive to length of pipe extension for $\lambda < 0.35$, where λ is the bend characteristic parameter, and generally

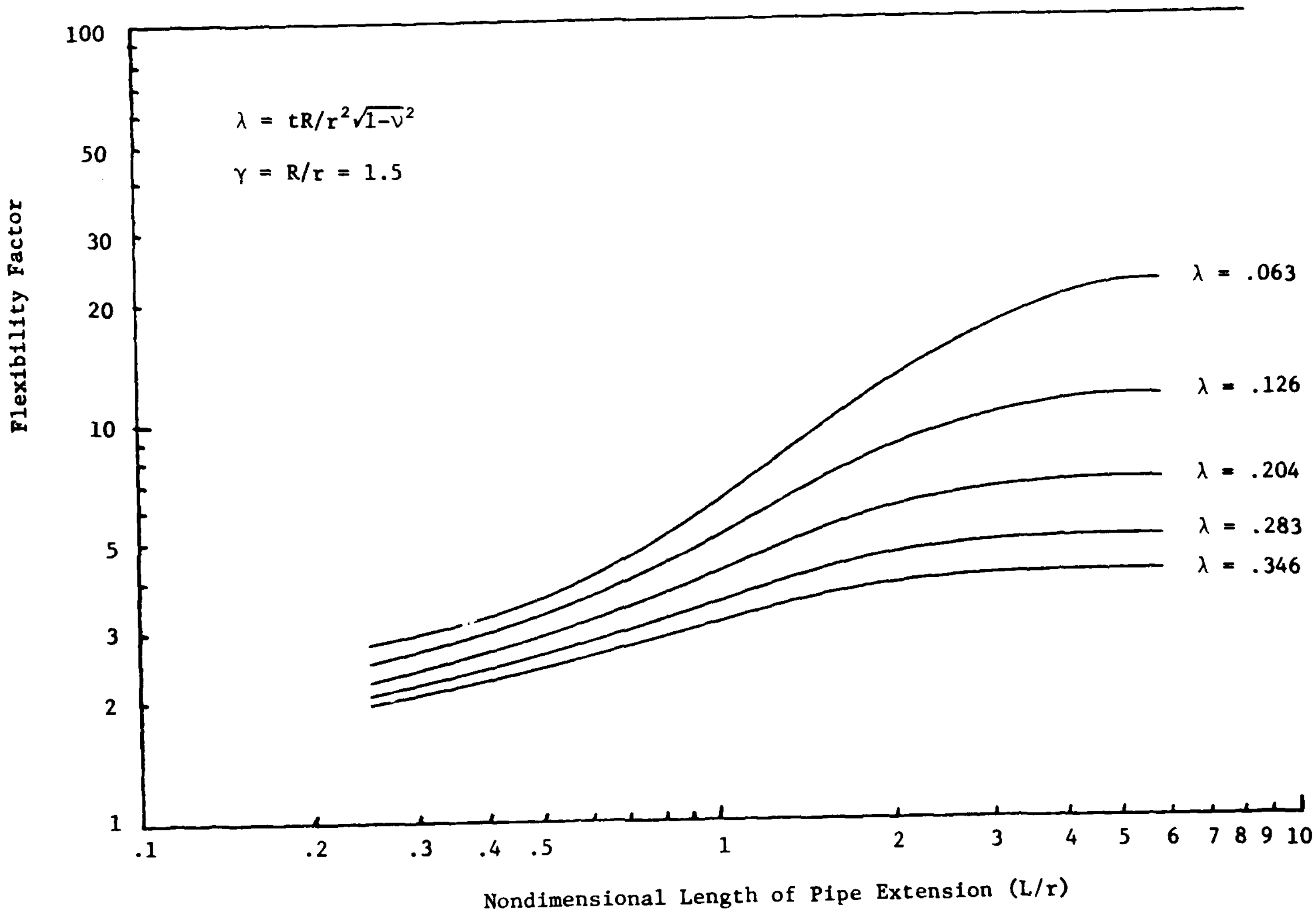

Figure 5 - Sensitivity of Flexibility Factors to Length of Pipe Extensions (γ = 1.5)

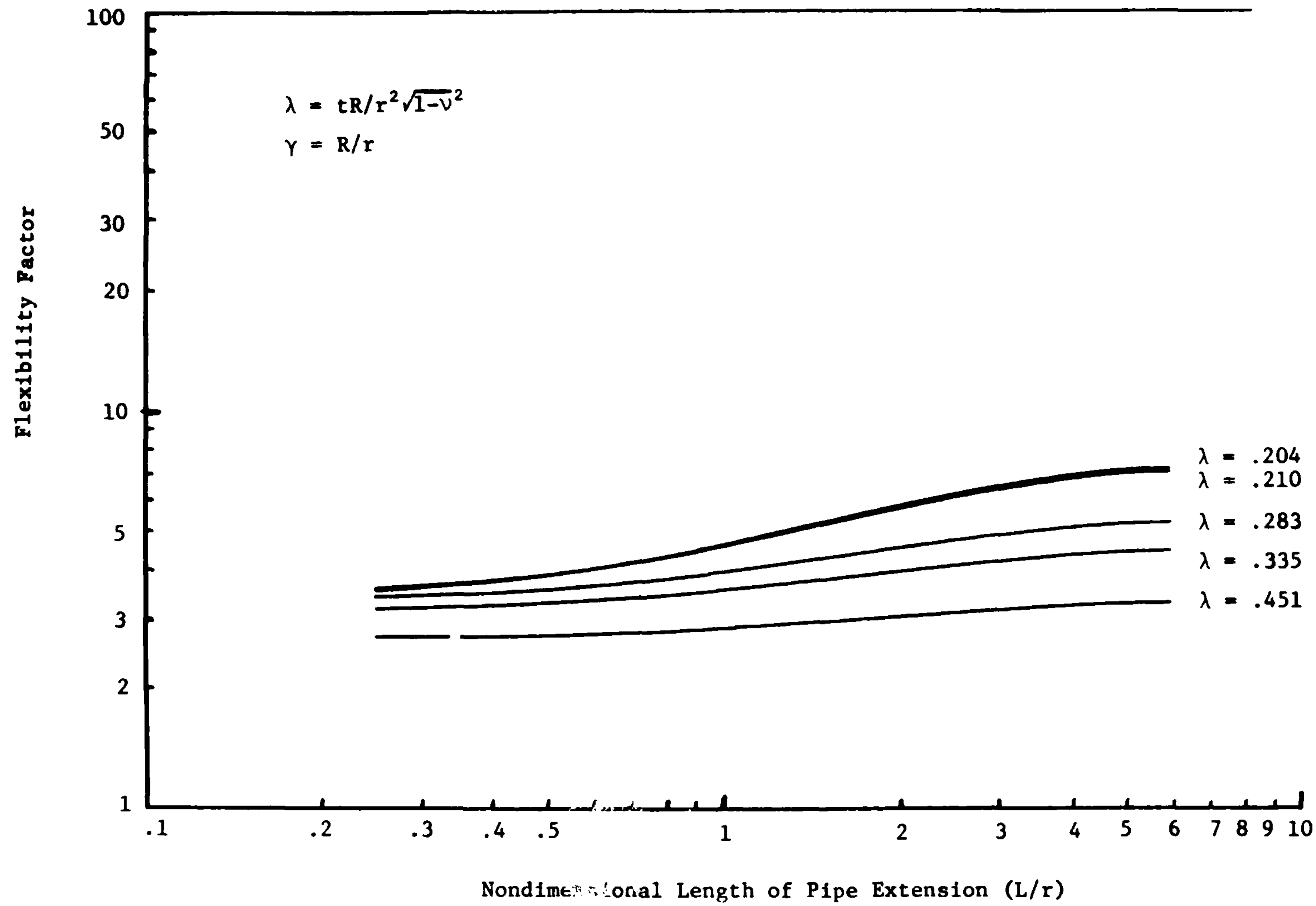

Figure 6 - Sensitivity of Flexibility Factors to Length of Pipe Extensions (t/r = 0.04)

TABLE 4 - FLEXIBILITY FACTORS FOR ELBOWS HAVING EQUAL LENGTH STRAIGHT PIPE EXTENSIONS OF LENGTH THREE DIAMETERS

	λ	γ	NASTRAN (Moment)		NASTRAN (Force)		ELBOW	ASME Code
			Inplane	Out-of-Plane	Inplane	Out-of-Plane		
1	.063	1.50	21.8	9.84	20.9	10.1	27.5	27.5
2	.126	1.50	11.3	5.44	10.8	5.47	13.8	13.7
3	.126	3.00	11.3	5.86	10.7	5.85	13.8	13.7
4	.204	1.50	6.98	3.62	6.63	3.58	8.49	8.48
5	.204	4.88	7.18	4.09	6.63	3.94	8.49	8.48
6	.210	5.00	7.03	4.02	6.54	3.91	8.27	8.24
7	.283	1.50	5.04	2.79	4.76	2.71	6.09	6.11
8	.283	6.75	5.28	3.23	4.80	3.04	6.09	6.11
9	.335	8.00	4.47	2.85	4.05	2.67	5.09	5.16
10	.346	1.50	4.14	2.41	3.89	2.31	4.92	5.00
11	.451	10.8	3.32	2.31	2.95	2.09	3.69	3.84
12	.451	1.80	3.19	2.05	2.91	1.92	3.69	3.84
13	.600	3.00	2.48	1.82	2.27	1.70	2.71	2.88
14	.700	3.00	2.15	1.67	1.97	1.55	2.32	2.47
15	.973	10.2	1.65	1.48	1.47	1.33	1.73	1.78
16	.975	3.00	1.69	1.46	1.59	1.37	1.73	1.77
17	1.50	9.49	1.30	1.31	1.15	1.16	1.32	1.15
18	1.50	3.90	1.35	1.31	1.23	1.20	1.32	1.15
19	2.39	8.00	1.14	1.23	1.02	1.10	1.13	1.00
20	2.39	5.15	1.17	1.24	1.07	1.13	1.13	1.00
21	3.01	5.75	1.14	1.23	1.00	1.08	1.08	1.00

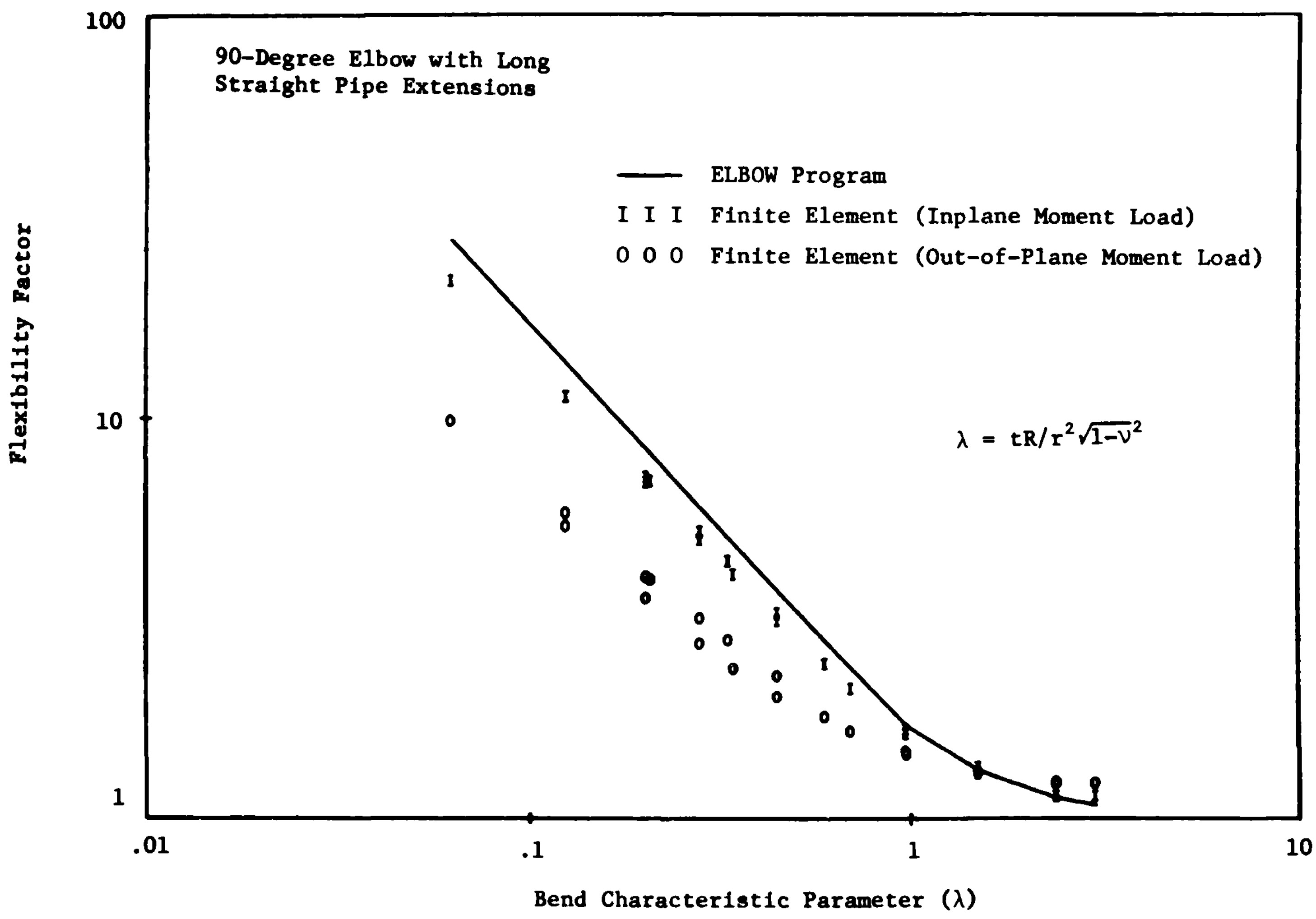

Figure 7 - Comparison of Finite Element and ELBOW Flexibility Factors

insensitive for $\lambda > 0.35$. The critical length of straight pipe extension (defined as the minimum length of pipe for which the stress index is insensitive to further increases in pipe length) is about one pipe diameter for $\lambda < 0.35$ and three diameters for $\lambda > 0.35$.

For elbows loaded by internal pressure only, the stress indices are insensitive to the length of straight pipe extension. The theoretical predictions by the classical relation given in the Kellogg book for this case are excellent.

Flexibility factors exhibit a sensitivity to length of straight pipe extension over the entire range of elbow parameters considered. Flexibility factors for out-of-plane loading differ considerably from those for inplane loading.

The fast-running computer program ELBOW, which idealizes elbows as sections of tori, is generally not adequate for predicting stress indices or flexibility factors for 90-degree elbows with straight pipe extensions.

RECOMMENDATIONS

Although a large quantity of useful data has been compiled in this report, the report is viewed as interim, since the information is not in a form to be easily used by a piping system designer. We therefore recommend that the data presented here be analyzed further to obtain usable design equations, formulas, or curves for 90-degree elbows with straight pipe extensions.

Since not all elbows used in naval designs are 90-degree elbows, we recommend that other bend angles (e.g., 45 degrees) be investigated. Also, future parametric studies would be less expensive if a more restrictive

selection of elbows is made.

The presence of an internal pressure preload is known to have a significant effect on moment stress indices and flexibility factors. A finite element procedure for this combined loading case, which is mathematically nonlinear, needs to be formulated and validated. Following such validation, the various parametric studies discussed above (for which the internal pressure is zero) should be repeated for several typical nonzero pressure levels.

ACKNOWLEDGMENTS

We acknowledge with pleasure the fruitful discussions held with Mr. L.M. Kaldor and Dr. Y.P. Lu of the Machinery Silencing Division (Code 274).

APPENDIX

Tables of Finite Element Results for 90-Degree Elbows with Straight Pipe Extensions

TABLE A1 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 1

(λ = .063, γ = 1.50)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
1A	12.	12.	10.3	10.2	1.93	22.1	10.0	21.6	10.1
1B	8.	8.	10.3	10.2	1.93	22.1	9.97	21.4	10.2
1C	6.	6.	10.2	10.0	1.93	21.8	9.84	20.9	10.1
1D	4.	4.	9.58	9.34	1.93	20.0	9.02	18.9	9.34
1E	2.	2.	6.35	6.05	1.93	12.4	5.92	11.1	6.07
1F	1.	1.	3.42	3.18	1.91	6.24	3.38	5.11	3.24
1G	.5	.5	2.43	2.66	1.87	3.66	2.31	2.59	1.89
1H	.25	.25	2.49	2.78	1.87	2.83	2.00	1.58	1.22
1I	4.	6.	9.90	9.67	1.93	20.9	9.24	19.8	9.57
1J	2.	6.	8.10	7.72	1.93	16.7	7.03	15.0	7.32
1K	1.	6.	5.85	5.41	1.92	11.8	4.80	9.95	4.90
1L	.5	6.	4.38	3.95	1.91	8.75	3.56	6.85	3.44
1M	.25	6.	3.65	3.44	1.93	5.92	3.11	5.29	2.65

ELBOW Stress Index (Moment) = 14.5
ELBOW Flexibility Factor = 27.5
ASME Code Flexibility Factor = 27.5
Kellogg Stress Index (Pressure) = 2.00

TABLE A2 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 2

(λ = .126, γ = 1.50)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
2A	6.	6.	6.55	6.41	1.89	11.3	5.44	10.8	5.47
2B	4.	4.	6.44	6.27	1.88	11.0	5.31	10.3	5.36
2C	2.	2.	5.19	4.94	1.88	8.47	4.24	7.53	4.22
2D	1.	1.	3.30	3.42	1.86	5.12	2.86	4.15	2.62
2E	.5	.5	2.88	3.01	1.87	3.33	2.12	2.30	1.59
2F	.25	.25	3.02	3.22	1.90	2.53	1.83	1.42	1.07

ELBOW Stress Index (Moment) = 9.28
ELBOW Flexibility Factor = 13.8
ASME Code Flexibility Factor = 13.7
Kellogg Stress Index (Pressure) = 2.00

TABLE A3 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 3

(λ = .126, γ = 3.00)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
3A	8.	8.	7.16	7.02	1.23	11.4	5.92	10.9	5.91
3B	6.	6.	7.13	6.96	1.23	11.3	5.86	10.7	5.85
3C	4.	4.	6.82	6.61	1.23	10.7	5.53	9.90	5.55
3D	2.	2.	5.21	4.95	1.23	7.91	4.24	7.03	4.27
3E	1.	1.	3.43	3.19	1.24	5.25	3.05	4.44	2.99
3F	.5	.5	2.42	2.29	1.24	3.84	2.42	3.11	2.28
3G	.25	.25	2.19	2.26	1.25	3.34	2.21	2.59	1.94
3H	4.	6.	6.98	6.78	1.23	11.0	5.55	10.3	5.60
3I	2.	6.	6.19	6.03	1.23	9.71	4.52	8.89	4.77
3J	1.	6.	5.31	5.25	1.24	8.39	3.61	7.61	4.05
3K	.5	6.	4.73	4.80	1.24	7.53	3.11	6.85	3.67
3L	.25	6.	4.42	4.57	1.24	7.12	2.93	6.41	3.47

ELBOW Stress Index (Moment) = 8.56
ELBOW Flexibility Factor = 13.8
ASME Code Flexibility Factor = 13.7
Kellogg Stress Index (Pressure) = 1.25

TABLE A4 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 4

(λ = .204, γ = 1.50)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
4A	8.	8.	4.62	4.55	1.82	6.98	3.62	6.72	3.59
4B	6.	6.	4.61	4.52	1.82	6.98	3.62	6.63	3.58
4C	4.	4.	4.59	4.45	1.82	6.93	3.59	6.44	3.53
4D	2.	2.	4.10	3.99	1.81	6.01	3.20	5.29	3.06
4E	1.	1.	3.32	3.53	1.80	4.19	2.44	3.34	2.12
4F	.5	.5	3.21	3.38	1.85	2.95	1.93	2.00	1.34
4G	.25	.25	3.39	3.65	1.84	2.27	1.68	1.29	.943
4H	2.	4.	4.34	4.11	1.81	6.47	3.31	5.70	3.18
4I	1.	4.	3.66	3.85	1.81	5.42	2.75	4.41	2.48
4J	.5	4.	3.26	3.61	1.83	4.49	2.32	3.25	1.81
4K	.25	4.	3.08	3.52	1.83	3.92	2.08	2.59	1.44

ELBOW Stress Index (Moment) = 6.82
ELBOW Flexibility Factor = 8.49
ASME Code Flexibility Factor = 8.48
Kellogg Stress Index (Pressure) = 2.00

TABLE A5 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 5

(λ = .204, γ = 4.88)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
5A	6.	6.	5.39	5.21	1.12	7.18	4.09	6.63	3.94
5B	4.	4.	5.33	5.14	1.11	6.90	3.94	6.28	3.82
5C	2.	2.	4.76	4.52	1.12	5.77	3.38	5.05	3.29
5D	1.	1.	4.00	3.80	1.12	4.62	2.84	3.95	2.78
5E	.5	.5	3.43	3.24	1.12	3.91	2.50	3.26	2.42
5F	.25	.25	3.10	2.93	1.12	3.61	2.37	2.96	2.24

ELBOW Stress Index (Moment) = 5.98
ELBOW Flexibility Factor = 8.49
ASME Code Flexibility Factor = 8.48
Kellogg Stress Index (Pressure) = 1.13

TABLE A6 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 6

(λ = .210, γ = 5.00)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
6A	8.	8.	5.30	5.13	1.12	7.07	4.05	6.57	3.89
6B	6.	6.	5.31	5.18	1.12	7.03	4.02	6.54	3.91
6C	4.	4.	5.26	5.07	1.12	6.76	3.87	6.14	3.75
6D	2.	2.	4.73	4.49	1.12	5.68	3.34	4.96	3.24
6E	1.	1.	4.02	3.81	1.12	4.59	2.82	3.91	2.76
6F	.5	.5	3.48	3.30	1.12	3.90	2.50	3.26	2.43
6G	.25	.25	3.16	3.00	1.11	3.57	2.38	2.92	2.25

ELBOW Stress Index (Moment) = 5.87
ELBOW Flexibility Factor = 8.27
ASME Code Flexibility Factor = 8.24
Kellogg Stress Index (Pressure) = 1.13

TABLE A7 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 7

(λ = .283, γ = 1.50)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
7A	6.	6.	3.62	3.68	1.77	5.04	2.79	4.76	2.71
7B	4.	4.	3.62	3.71	1.76	5.03	2.78	4.65	2.68
7C	2.	2.	3.66	3.71	1.76	4.62	2.60	4.02	2.41
7D	1.	1.	3.36	3.53	1.77	3.54	2.15	2.78	1.78
7E	.5	.5	3.35	3.54	1.81	2.65	1.79	1.74	1.16
7F	.25	.25	3.46	3.74	1.73	2.10	1.58	1.22	.869

ELBOW Stress Index (Moment) = 5.53
ELBOW Flexibility Factor = 6.09
ASME Code Flexibility Factor = 6.11
Kellogg Stress Index (Pressure) = 2.00

TABLE A8 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 8

(λ = .283, γ = 6.75)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
8A	6.	6.	4.39	4.26	1.08	5.28	3.23	4.80	3.04
8B	4.	4.	4.41	4.25	1.08	5.13	3.14	4.57	2.95
8C	2.	2.	4.26	4.10	1.08	4.55	2.85	3.95	2.71
8D	1.	1.	4.00	3.84	1.08	3.97	2.57	3.39	2.47
8E	.5	.5	3.76	3.61	1.08	3.60	2.39	3.03	2.29
8F	.25	.25	3.60	3.47	1.08	3.46	2.33	2.88	2.20

ELBOW Stress Index (Moment) = 4.70
ELBOW Flexibility Factor = 6.09
ASME Code Flexibility Factor = 6.11
Kellogg Stress Index (Pressure) = 1.09

TABLE A9 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 9

(λ = .335, γ = 8.00)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
9A	6.	6.	3.90	3.81	1.07	4.47	2.85	4.05	2.67
9B	4.	4.	3.93	3.82	1.07	4.36	2.79	3.88	2.60
9C	2.	2.	3.90	3.74	1.07	3.96	2.58	3.40	2.40
9D	1.	1.	3.78	3.66	1.07	3.57	2.39	3.05	2.26
9E	.5	.5	3.66	3.53	1.07	3.32	2.27	2.79	2.14
9F	.25	.25	3.58	3.45	1.07	3.21	2.23	2.66	2.08

ELBOW Stress Index (Moment) = 4.14
ELBOW Flexibility Factor = 5.09
ASME Code Flexibility Factor = 5.16
Kellogg Stress Index (Pressure) = 1.07

TABLE A10 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 10

($\lambda = .346$, $\gamma = 1.50$)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
10A	6.	6.	3.39	3.45	1.74	4.14	2.41	3.89	2.31
10B	4.	4.	3.39	3.48	1.74	4.13	2.40	3.80	2.27
10C	2.	2.	3.40	3.53	1.73	3.90	2.30	3.37	2.07
10D	1.	1.	3.36	3.49	1.76	3.16	1.98	2.45	1.58
10E	.5	.5	3.37	3.57	1.78	2.45	1.69	1.61	1.08
10F	.25	.25	3.41	3.69	1.66	1.99	1.51	1.15	.810
10G	4.	6.	3.39	3.48	1.74	4.14	2.40	3.80	2.27
10H	2.	6.	3.39	3.53	1.73	4.02	2.32	3.48	2.10
10I	1.	6.	3.32	3.53	1.75	3.62	2.10	2.84	1.71
10J	.5	6.	3.19	3.50	1.76	3.17	1.88	2.18	1.29
10K	.25	6.	3.07	3.48	1.69	2.84	1.73	1.80	1.06

ELBOW Stress Index (Moment) = 4.88
ELBOW Flexibility Factor = 4.92
ASME Code Flexibility Factor = 5.00
Kellogg Stress Index (Pressure) = 2.00

TABLE A11 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 11

(λ = .451, γ = 10.8)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
11A	6.	6.	3.10	3.02	1.04	3.32	2.31	2.95	2.09
11B	4.	4.	3.13	3.05	1.04	3.27	2.28	2.86	2.05
11C	2.	2.	3.17	3.08	1.04	3.06	2.17	2.62	1.96
11D	1.	1.	3.18	3.11	1.04	2.87	2.08	2.44	1.89
11E	.5	.5	3.17	3.08	1.04	2.75	2.02	2.31	1.45
11F	.25	.25	3.15	3.05	1.04	2.73	2.00	2.26	1.80
11G	4.	6.	3.11	3.03	1.04	3.30	2.28	2.89	2.06
11H	2.	6.	3.13	3.03	1.04	3.19	2.17	2.78	1.98
11I	1.	6.	3.14	3.04	1.04	3.10	2.08	2.72	1.94
11J	.5	6.	3.13	3.02	1.04	3.04	2.03	2.66	1.90
11K	.25	6.	3.12	2.99	1.04	3.03	2.02	2.62	1.87

ELBOW Stress Index (Moment) = 3.29
ELBOW Flexibility Factor = 3.69
ASME Code Flexibility Factor = 3.84
Kellogg Stress Index (Pressure) = 1.05

TABLE A12 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 12

(λ = .451, γ = 1.80)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
12A	6.	6.	2.77	2.83	1.56	3.19	2.05	2.99	1.95
12B	4.	4.	2.77	2.85	1.56	3.19	2.05	2.91	1.92
12C	2.	2.	2.80	2.91	1.56	3.06	1.99	2.64	1.75
12D	1.	1.	2.85	2.96	1.57	2.64	1.81	2.08	1.47
12E	.5	.5	2.90	3.05	1.62	2.21	1.63	1.55	1.14
12F	.25	.25	2.94	3.15	1.60	1.91	1.50	1.24	.968

ELBOW Stress Index (Moment) = 3.94
ELBOW Flexibility Factor = 3.69
ASME Code Flexibility Factor = 3.84
Kellogg Stress Index (Pressure) = 1.63

TABLE A13 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 13

(λ = .600, γ = 3.00)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
13A	6.	6.	2.43	2.32	1.25	2.48	1.82	2.27	1.70
13B	4.	4.	2.43	2.29	1.25	2.47	1.82	2.21	1.67
13C	2.	2.	2.39	2.22	1.25	2.41	1.79	2.05	1.58
13D	1.	1.	2.21	2.07	1.25	2.21	1.70	1.78	1.43
13E	.5	.5	2.10	2.20	1.25	2.03	1.62	1.63	1.35
13F	.25	.25	2.15	2.20	1.25	1.91	1.57	1.47	1.26

ELBOW Stress Index (Moment) = 2.93
ELBOW Flexibility Factor = 2.71
ASME Code Flexibility Factor = 2.88
Kellogg Stress Index (Pressure) = 1.25

TABLE A14 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 14

(λ = .700, γ = 3.00)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
14A	6.	6.	2.12	2.04	1.25	2.15	1.67	1.97	1.55
14B	4.	4.	2.12	2.03	1.25	2.15	1.67	1.93	1.53
14C	2.	2.	2.10	1.97	1.25	2.11	1.65	1.80	1.44
14D	1.	1.	1.98	1.96	1.25	1.98	1.59	1.60	1.30
14E	.5	.5	2.03	2.02	1.25	1.85	1.53	1.41	1.20
14F	.25	.25	2.08	2.02	1.25	1.75	1.49	1.28	1.12

ELBOW Stress Index (Moment) = 2.61
ELBOW Flexibility Factor = 2.32
ASME Code Flexibility Factor = 2.47
Kellogg Stress Index (Pressure) = 1.25

TABLE A15 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 15

($\lambda = .973$, $\gamma = 10.2$)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
15A	6.	6.	1.67	1.71	1.05	1.65	1.48	1.47	1.33
15B	4.	4.	1.67	1.71	1.05	1.65	1.48	1.43	1.30
15C	2.	2.	1.68	1.73	1.05	1.62	1.47	1.38	1.26
15D	1.	1.	1.70	1.75	1.05	1.58	1.45	1.32	1.23
15E	.5	.5	1.72	1.78	1.05	1.57	1.44	1.29	1.21
15F	.25	.25	1.72	1.79	1.05	1.57	1.44	1.28	1.20

ELBOW Stress Index (Moment) = 2.00
ELBOW Flexibility Factor = 1.73
ASME Code Flexibility Factor = 1.78
Kellogg Stress Index (Pressure) = 1.05

TABLE A16 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 16

(λ = .975, γ = 3.00)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
16A	8.	8.	1.76	1.79	1.26	1.69	1.46	1.59	1.37
16B	6.	6.	1.76	1.80	1.26	1.69	1.46	1.56	1.35
16C	4.	4.	1.76	1.81	1.26	1.69	1.46	1.52	1.32
16D	2.	2.	1.78	1.86	1.26	1.67	1.45	1.43	1.25
16E	1.	1.	1.84	1.94	1.26	1.63	1.43	1.31	1.16
16F	.5	.5	1.90	2.02	1.27	1.57	1.40	1.20	1.08
16G	.25	.25	1.95	2.08	1.28	1.51	1.37	1.14	1.05

ELBOW Stress Index (Moment) = 2.13
ELBOW Flexibility Factor = 1.73
ASME Code Flexibility Factor = 1.77
Kellogg Stress Index (Pressure) = 1.25

TABLE A17 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 17

(λ = 1.50, γ = 9.49)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
17A	6.	6.	1.47	1.45	1.06	1.30	1.31	1.15	1.16
17B	4.	4.	1.47	1.46	1.06	1.30	1.31	1.13	1.14
17C	2.	2.	1.47	1.47	1.06	1.29	1.30	1.09	1.10
17D	1.	1.	1.47	1.48	1.06	1.28	1.30	1.06	1.08
17E	.5	.5	1.4[illegible]	1.49	1.06	1.28	1.30	1.04	1.06
17F	.25	.25	1.47	1.49	1.06	1.28	1.30	1.04	1.05

ELBOW Stress Index (Moment) = 1.71
ELBOW Flexibility Factor = 1.32
ASME Code Flexibility Factor = 1.15
Kellogg Stress Index (Pressure) = 1.06

TABLE A18 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 18

(λ = 1.50, γ = 3.90)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
18A	6.	6.	1.49	1.54	1.19	1.35	1.31	1.23	1.20
18B	4.	4.	1.49	1.55	1.19	1.35	1.31	1.20	1.17
18C	2.	2.	1.50	1.58	1.19	1.34	1.31	1.14	1.11
18D	1.	1.	1.52	1.62	1.19	1.33	1.31	1.08	1.06
18E	.5	.5	1.55	1.66	1.19	1.32	1.30	1.04	1.02
18F	.25	.25	1.57	1.69	1.19	1.30	1.29	1.01	1.00

ELBOW Stress Index (Moment) = 1.75
ELBOW Flexibility Factor = 1.32
ASME Code Flexibility Factor = 1.15
Kellogg Stress Index (Pressure) = 1.17

TABLE A19 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 19

(λ = 2.39, γ = 8.00)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
19A	6.	6.	1.27	1.25	1.07	1.14	1.23	1.02	1.10
19B	4.	4.	1.27	1.26	1.07	1.14	1.23	.996	1.07
19C	2.	2.	1.27	1.27	1.07	1.14	1.23	.961	1.04
19D	1.	1.	1.27	1.28	1.07	1.14	1.23	.936	1.01
19E	.5	.5	1.27	1.29	1.07	1.14	1.23	.924	.991
19F	.25	.25	1.27	1.29	1.07	1.14	1.23	.917	.986

ELBOW Stress Index (Moment) = 1.52
ELBOW Flexibility Factor = 1.13
ASME Code Flexibility Factor = 1.00
Kellogg Stress Index (Pressure) = 1.07

TABLE A20 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 20

(λ = 2.39, γ = 5.15)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
20A	6.	6.	1.32	1.37	1.14	1.17	1.24	1.07	1.13
20B	4.	4.	1.32	1.37	1.14	1.17	1.24	1.04	1.10
20C	2.	2.	1.32	1.39	1.14	1.17	1.24	.995	1.05
20D	1.	1.	1.33	1.41	1.14	1.18	1.24	.960	1.01
20E	.5	.5	1.33	1.43	1.14	1.18	1.24	.939	1.00
20F	.25	.25	1.34	1.44	1.14	1.17	1.24	.926	.977

ELBOW Stress Index (Moment) = 1.52
ELBOW Flexibility Factor = 1.13
ASME Code Flexibility Factor = 1.00
Kellogg Stress Index (Pressure) = 1.12

TABLE A21 - SUMMARY OF FINITE ELEMENT RESULTS FOR ELBOW 21

($\lambda = 3.01$, $\gamma = 5.75$)

Elbow ID	LA/r	LB/r	NASTRAN Stress Index			NASTRAN Flexibility Factor (Moment)		NASTRAN Flexibility Factor (Force)	
			Moment	Force	Pressure	Inplane	Out-of-Plane	Inplane	Out-of-Plane
21A	4.	4.	1.28	1.33	1.12	1.14	1.23	1.00	1.08
21B	2.	2.	1.28	1.34	1.12	1.14	1.23	.962	1.04
21C	1.	1.	1.28	1.36	1.12	1.14	1.23	.932	1.00
21D	.5	.5	1.28	1.37	1.12	1.14	1.23	.915	.981
21E	.25	.25	1.28	1.38	1.12	1.13	1.23	.904	.973
21F	.5	1.	1.28	1.37	1.12	1.14	1.23	.915	.981
21G	.25	1.	1.28	1.37	1.12	1.14	1.23	.908	.973

ELBOW Stress Index (Moment) = 1.45
ELBOW Flexibility Factor = 1.08
ASME Code Flexibility Factor = 1.00
Kellogg Stress Index (Pressure) = 1.11

REFERENCES

1. Marcus, M.S., and G.C. Everstine, "Finite Element Analysis of Pipe Elbows," Report CMLD-79/15, David W. Taylor Naval Ship Research and Development Center, Bethesda, Maryland (Feb 1980).

2. Quezon, A.J., G.C. Everstine, and M.E. Golden, "Finite Element Analysis of Piping Tees," Report CMLD-80/11, David W. Taylor Naval Ship Research and Development Center, Bethesda, Maryland (June 1980).

3. Quezon, A.J., and G.C. Everstine, "Comparison of Finite Element Analyses of a Piping Tee Using NASTRAN and CORTES/SA," Ninth NASTRAN Users' Colloquium, NASA CP-2151, Washington, D.C., pp. 224-242 (Oct 1980).

4. Dodge, W.G., and S.E. Moore, "Stress Indices and Flexibility Factors for Moment Loadings on Elbows and Curved Pipe," Report ORNL-TM-3658, Oak Ridge National Laboratory, Oak Ridge, Tennessee (March 1972).

5. The M.W. Kellogg Company, Design of Piping Systems, Second Edition, John Wiley and Sons, Inc., New York (1964).

6. Dodge, W.G., and S.E. Moore, "ELBOW: A Fortran Program for the Calculation of Stresses, Stress Indices, and Flexibility Factors for Elbows and Curved Pipe," Report ORNL-TM-4098, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Apr 1973).

7. American Society of Mechanical Engineers, "ASME Boiler and Pressure Vessel Code. Section III, Rules for Construction of Nuclear Power Plant Components" (1980 edition).

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